ENTROPY ECONOMY SYSTEM

A new economic paradigm.

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Entropy Economy System is the first part of the Pale Blueprint initiative. Its objective is to introduce the formalism and the theory on which the whole project is based. For better readability, the axioms, theorems, and examples are placed in blue frames and definitions are indicated by terms in italics.

Contents

1 Thermodynamics

Thermodynamics is a branch of physics that studies the thermal behavior of bodies, energy and its transformations. This first section is intended to remind the main tools and concepts introduced by thermodynamics in order to lay the foundations for the development of Entropy Economy System. Some project-specific conventions are also introduced here.

1.1 Physical systems

A *physical system* is a portion of the physical universe chosen for analysis. Everything outside the system is known as the *surroundings*. A proper definition of a physical system allows the study of characteristics associated with it. The three categories of systems below are usually distinguished.

- An *open system* can exchange matter and energy with its surroundings.
- A *closed system* can exchange energy, but no matter with its surroundings.
- An *isolated system* can't exchange neither energy nor matter with its surroundings.

A certain proximity in the manipulation of mathematical sets and physical systems will be maintained. This proximity allows a better mathematical formulation of the principles developed. It is recalled that from a mathematical point of view a set is a collection of distinct objects.

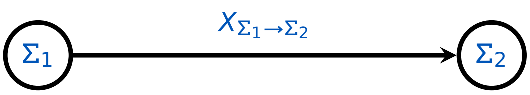
- Systems will be represented as sets of objects called *atomic systems*.
  - Atomic systems are indivisible and their position in space can be defined. [A.1]
  - If Σ is a system, $\sigma \in \Sigma$ is an atomic system while $\Pi \subset \Sigma$ is a system.
  - Atomic systems are noted with lower case and systems with upper case.
- Operations on mathematical sets are extended to physical systems.
  - When Σ denotes a system, $\overline{\Sigma}$ denotes its surroundings.
  - $\cup$ translates the union of systems. For instance: $\{\sigma_1, \sigma_2, \sigma_3\} \cup \{\sigma_2, \sigma_3, \sigma_4\} = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$.
  - $\cap$ translates the intersection of systems. For instance: $\{\sigma_1, \sigma_2, \sigma_3\} \cap \{\sigma_2, \sigma_3, \sigma_4\} = \{\sigma_2, \sigma_3\}$.

1.2 Physical quantities

A *physical quantity* is a property of a system that can be quantified by measuring and expressed as the combination of a *magnitude* and a *unit*. A *state function* is a physical quantity whose value does not depend on the path taken to reach that specific value. A *path function* is a physical quantity that depends on the path taken to reach that specific value.

An *intensive property* does not depend on the system size or the amount of material it contains. An *extensive property* is additive for subsystems because it increases and decreases as they grow larger and smaller, respectively.

- The infinitesimal variations are noted differently depending on the physical quantity studied:
  - δ denotes an inexact differential (for path functions).
  - d denotes an exact differential (for state functions).
- Systems and quantities exchanged can be represented by diagrams like the following one:



- The sign of the quantities exchanged determine whether they are received or supplied by a system:
  - If $X_{\Sigma_1 \to \Sigma_2} > 0$, then Σ_2 receives $|X_{\Sigma_1 \to \Sigma_2}|$.
  - If $X_{\Sigma_1 \to \Sigma_2} < 0$, then Σ_2 supplies $|X_{\Sigma_1 \to \Sigma_2}|$.
- $X_{\Sigma_1 \to \Sigma_2} = -X_{\Sigma_2 \to \Sigma_1}$

1.3 First law of thermodynamics

The *internal energy* of a system is an extensive state function whose unit is the joule (J). It is defined as the energy contained within the system.

The first law of thermodynamics defines the variation of internal energy in a closed system as the sum of the heat added to the system and the work done by the surroundings on the system.

Axiom

Hypotheses

- Σ a closed system
- U_Σ the internal energy of Σ (J)
- $W_{\overline{\Sigma}\to\Sigma}$ the amount of work exchanged (J)
- $Q_{\overline{\Sigma}\to\Sigma}$ the amount of heat exchanged (J)

Conclusion

- $dU_\Sigma = \delta W_{\overline{\Sigma}\to\Sigma} + \delta Q_{\overline{\Sigma}\to\Sigma}$ **(1.3.1)**

Take-home message:

- The total energy of an isolated system is constant.
- Energy can be transformed from one form to another, but can be neither created nor destroyed.

1.4 Second law of thermodynamics

The *entropy* of a system is an extensive state function whose unit is the joule per kelvin (J/K). It can be interpreted as a measure of the degree of disorder at the microscopic level. The higher the entropy of the system, the less its elements are ordered, linked together, capable of producing mechanical effects, and the greater is the share of unusable energy for obtaining work.

The second law of thermodynamics states that the total entropy of an isolated system can never decrease over time, and is constant if and only if all processes are *reversible*.

Axiom

Hypotheses

- Σ a closed system
- S_Σ the entropy of Σ (J/K)
- S_Σ^C the created entropy in Σ (J/K)
- $Q_{\overline{\Sigma}\to\Sigma}$ the amount of heat exchanged (J)
- $T_{\overline{\Sigma}}$ the temperature of $\overline{\Sigma}$ (K)

Conclusion

- $dS_\Sigma = \delta S_\Sigma^C + \dfrac{\delta Q_{\overline{\Sigma}\to\Sigma}}{T_{\overline{\Sigma}}}$ **(1.4.1)**
- $\delta S_\Sigma^C \geq 0$ **(1.4.2)**

Take-home message:

- The total entropy of an isolated system can never decrease over time.
- Isolated systems spontaneously evolve towards thermodynamic equilibrium.

2 Principles

This section shows the construction of the theoretical elements proposed by Entropy Economy System. The hypotheses used here are the two laws of thermodynamics described in the previous section and an additional axiom about the sustainability of human societies proposed in subsection 2.2.

This first layer of theory leads to three general principles which will give the main guidelines of the project. In particular with regard to the importance of energy in this new economic paradigm. However, it will be necessary to deepen these three principles after seeing how to describe the environment in which they apply because they do not take into account the problems posed by transfers between systems (possibilities, losses, spatial arrangement, grid) and do not distinguish the different forms of energy either (conversion losses).

2.1 General equation

The `Helmholtz free energy` is an extensive state function whose unit is the joule (J). It measures the useful work obtainable from a closed thermodynamic system at a constant temperature and volume.

By its definition, Helmholtz free energy is strongly linked to the entropy of the system. It is established here this relationship in a general case.

Theorem

Hypotheses

- Σ a closed system
- U_Σ the internal energy of Σ (J)
- $T_{\overline{\Sigma}}$ the temperature of $\overline{\Sigma}$ (K)
- S_Σ^C the created entropy in Σ (J/K)
- S_Σ the entropy of Σ (J/K)
- $F_\Sigma = U_\Sigma - T_{\overline{\Sigma}} S_\Sigma$ the Helmholtz free energy of Σ (J)
- $W_{\overline{\Sigma}\to\Sigma}$ the amount of work exchanged (J)

Conclusion

- $dF_\Sigma = \delta W_{\overline{\Sigma}\to\Sigma} - T_{\overline{\Sigma}} \delta S_\Sigma^C - S_\Sigma dT_{\overline{\Sigma}}$ **(2.1.1)**

Proof

- The differential of the Helmholtz free energy is:
  $dF_\Sigma = dU_\Sigma - S_\Sigma dT_{\overline{\Sigma}} - T_{\overline{\Sigma}} dS_\Sigma$
- With **(1.4.1)**:
  $\delta Q_{\overline{\Sigma}\to\Sigma} = T_{\overline{\Sigma}} \left(dS_\Sigma - \delta S_\Sigma^C \right)$
- By replacing $\delta Q_{\overline{\Sigma}\to\Sigma}$ in **(1.3.1)**:
  $dU_\Sigma = \delta W_{\overline{\Sigma}\to\Sigma} + T_{\overline{\Sigma}} \left(dS_\Sigma - \delta S_\Sigma^C \right)$
- By replacing dU_Σ in the expression of dF_Σ:
  $dF_\Sigma = \delta W_{\overline{\Sigma}\to\Sigma} + T_{\overline{\Sigma}} \left(dS_\Sigma - \delta S_\Sigma^C \right) - S_\Sigma dT_{\overline{\Sigma}} - T_{\overline{\Sigma}} dS_\Sigma$

Take-home message:

- Creation of entropy leads to a decrease in Helmholtz free energy.

2.2 Sustainable society

Here are proposed the conditions for obtaining a sustainable society. To formulate them mathematically it is necessary to consider the society as a physical system. Then it is defined the *vital system* of this society that corresponds to the parts ensuring functions that have been identified as vital ones.

Axiom

Hypotheses

- Λ the closed system corresponding to the infrastructure and the population of a society
- $\Omega \subset \Lambda$ the vital system of Λ
- F_Λ the Helmholtz free energy of Λ (J)
- $\forall \omega \in \Omega \quad F_\omega$ the Helmholtz free energy of ω (J)
- $T_{\overline{\Lambda}}$ the temperature of $\overline{\Lambda}$ (K)
- Λ is a sustainable society

Conclusion

- $dF_\Lambda = 0$ **(2.2.1)**
- $dT_{\overline{\Lambda}} = 0$ **(2.2.2)**
- $\forall \omega \in \Omega \quad dF_\omega = 0$ **(2.2.3)**

Take-home message:

- With **(2.2.1)** the sustainability of a society is translated into physics by a steady state. [A.2]
- With **(2.2.2)** it is suggested that the surroundings temperature must not vary significantly. [A.3]
- With **(2.2.3)** the reasoning is the same as for **(2.2.1)** but applied to each vital component. [A.2]

2.3 Balance

The principle of sustainability and the general equation established previously lead to an equilibrium relationship between the energy that society stores and the entropy it creates.

Theorem

Hypotheses

- Λ the closed system corresponding to a sustainable society
- $W_{\overline{\Lambda} \to \Lambda}$ the amount of work exchanged (J)
- $T_{\overline{\Lambda}}$ the temperature of $\overline{\Lambda}$ (K)
- S^C_Λ the created entropy in Λ (J/K)

Conclusion

- $\delta W_{\overline{\Lambda} \to \Lambda} = T_{\overline{\Lambda}} \delta S^C_\Lambda$ **(2.3.1)**

Proof

- By applying **(2.1.1)** to Λ we obtain:

  $dF_\Lambda = \delta W_{\overline{\Lambda} \to \Lambda} - T_{\overline{\Lambda}} \delta S^C_\Lambda - S_\Lambda dT_{\overline{\Lambda}}$
- By applying **(2.2.1)** and **(2.2.2)** we obtain:

  $0 = \delta W_{\overline{\Lambda} \to \Lambda} - T_{\overline{\Lambda}} \delta S^C_\Lambda$

Take-home message:

- The energy a sustainable society requires is proportionally related to the entropy it creates.
- Given that the available energy is a limited resource, entropy created must be regulated.
- $\delta W_{\overline{\Lambda}\to\Lambda} \geq T_{\overline{\Lambda}} \delta S^C_{\Lambda}$ is also suitable in the sense that:
    - $\delta W_{\overline{\Lambda}\to\Lambda}$ is the quantity received
    - $T_{\overline{\Lambda}} \delta S^C_{\Lambda}$ is the quantity used

2.4 Currency

According to the previous subsection, the entropy created must be regulated. But entropy is not an easy quantity to observe and quantify. A currency that translates this principle of economy of entropy is needed. The natural candidate for this new currency is the Helmholtz free energy.

Theorem

Hypotheses

- Λ the closed system corresponding to a sustainable society
- $\lambda \in \Lambda$ a component of society
- $W_{\overline{\lambda}\to\lambda}$ the amount of work exchanged (J)
- F_λ the Helmholtz free energy of λ (J)
- $T_{\overline{\lambda}} = T_{\overline{\Lambda}}$ the temperature of $\overline{\Lambda}$ (K) [A.3]
- S^C_λ the entropy created by λ (J/K)

Conclusion

- $W_{\overline{\lambda}\to\lambda} = 0 \Longrightarrow \Delta F_\lambda = -T_{\overline{\Lambda}} S^C_\lambda$ **(2.4.1)**

Proof

- By applying **(2.1.1)** to λ: [A.4]
  $dF_\lambda = \delta W_{\overline{\lambda}\to\lambda} - T_{\overline{\lambda}} \delta S^C_\lambda - S_\lambda dT_{\overline{\lambda}}$
- Considering that $T_{\overline{\lambda}} = T_{\overline{\Lambda}}$ and by applying **(2.2.2)**:
  $dF_\lambda = \delta W_{\overline{\lambda}\to\lambda} - T_{\overline{\Lambda}} \delta S^C_\lambda$
- By integrating:
  $\Delta F_\lambda = W_{\overline{\lambda}\to\lambda} - T_{\overline{\Lambda}} S^C_\lambda$

Take-home message:

- The assertion $W_{\overline{\lambda}\to\lambda} = 0$ specifies that exchanges with other systems are temporarily not taken into account. What is at stake here is to determine a way to quantify the intrinsic cost of the action. The wealth exchanges between the different systems will be deepened later.
- Energy is a currency consistent with the principle of regulation of entropy created because:
    - F_λ can be considered as the capital of λ.
    - An action results in a decrease in capital proportional to the entropy created according to **(2.4.1)**.

2.5 Distribution

If the vital system is not the empty set, since it is part of society, the entropy it creates is limited by the entropy that society can create. Therefore, the Helmholtz free energy that society can provide must be sufficient to power the vital system.

Theorem

Hypotheses

- Λ the closed system corresponding to a sustainable society
- $\Omega \subset \Lambda$ the vital system of Λ
- $T_{\overline{\Omega}} = T_{\overline{\Lambda}}$ the temperature of $\overline{\Lambda}$ (K) A.3
- $W_{\overline{\Lambda} \to \Lambda}$ the amount of work supplied to Λ (J)
- $W_{\overline{\Omega} \to \Omega}$ the amount of work supplied to Ω (J)

Conclusion

- $\delta W_{\overline{\Lambda} \to \Lambda} \geq \delta W_{\overline{\Omega} \to \Omega}$ **(2.5.1)**

Proof

- With $\Omega \subset \Lambda$ and by positivity and extensivity of the entropy created:
  $\delta S^C_\Lambda \geq \delta S^C_\Omega$
- With **(2.3.1)**:
  $$\frac{\delta W_{\overline{\Lambda} \to \Lambda}}{T_{\overline{\Lambda}}} = \delta S^C_\Lambda$$
- With **(2.2.3)**:
  $$dF_\Omega = \sum_{\omega \in \Omega} dF_\omega = 0$$
- With $T_{\overline{\Omega}} = T_{\overline{\Lambda}}$ and **(2.1.1)** applied to Ω:
  $$\frac{\delta W_{\overline{\Omega} \to \Omega}}{T_{\overline{\Lambda}}} = \delta S^C_\Omega$$
- Finally:
  $$\frac{\delta W_{\overline{\Lambda} \to \Lambda}}{T_{\overline{\Lambda}}} \geq \frac{\delta W_{\overline{\Omega} \to \Omega}}{T_{\overline{\Lambda}}}$$

Take-home message:

- The energy generated must be sufficient to power the vital components.

2.6 Synthesis

The theory of Entropy Economy System can be summarized in the following three principles.

Principle 1: **(2.3.1)** $\iff \left(\dot{W}_{\overline{\Lambda} \to \Lambda} = T_{\overline{\Lambda}} \dot{S}^C_\Lambda\right)$

The power required by society is proportional to the time derivative of the entropy created.

Principle 2: **(2.4.1)** $\iff \left(W_{\overline{\lambda} \to \lambda} = 0 \implies \Delta F_\lambda = -T_{\overline{\Lambda}} S^C_\lambda\right)$

The currency is the Helmholtz free energy.

Principle 3: **(2.5.1)** $\iff \left(\dot{W}_{\overline{\Lambda} \to \Lambda} \geq \dot{W}_{\overline{\Omega} \to \Omega}\right)$

The power supplied by society must adapt to the demand of the vital components.

3 Societies

This section introduces the formalism that will be used to describe the organization and functioning of a society Λ. Characteristics are expressed from a limited number of mathematical objects. These new tools will make it possible to propose concrete applications of the general principles established in the previous section.

3.1 Energies

Storing energy in one form or another requires different means from a practical point of view. Furthermore conversions create entropy. It is therefore appropriate to distinguish forms of energy. The mathematical object E_Λ called `energies` is defined for this purpose.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- E_Λ the energies of Λ

Proposals

- $n \in \mathbb{N}^*$ the number of forms of energy known in Λ
- $E_\Lambda = [\![1, n]\!]$ **(3.1.1)**

3.2 Systems

As in any physical problem, it is essential to properly define the interacting systems. The society Λ is therefore partitioned into subsystems. These subsystems can be partitioned again as many times as necessary. Thus the organization of the components of a society can be represented by a tree structure S_Λ called `systems`.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- S_Λ the systems of Λ

Proposals

- $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8\}$ contains 8 atomic systems
- $P(\Lambda) = \{\{\lambda_1, \lambda_3, \lambda_5, \lambda_7\}, \{\lambda_2, \lambda_4, \lambda_6, \lambda_8\}\}$ is an example of possible partitioning
- It is possible to go down further in the partitioning levels. For instance:
  - $P(\{\lambda_1, \lambda_3, \lambda_5, \lambda_7\}) = \{\{\lambda_1, \lambda_3\}, \{\lambda_5, \lambda_7\}\}$
  - $P(\{\lambda_2, \lambda_4, \lambda_6, \lambda_8\}) = \{\{\lambda_2, \lambda_4\}, \{\lambda_6, \lambda_8\}\}$
- All of the above information can be grouped in the partition tree S_Λ below:

$$\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8\}$$

$$\{\lambda_1, \lambda_3, \lambda_5, \lambda_7\} \qquad \{\lambda_2, \lambda_4, \lambda_6, \lambda_8\}$$

$$\{\lambda_1, \lambda_3\} \quad \{\lambda_5, \lambda_7\} \quad \{\lambda_6, \lambda_8\} \quad \{\lambda_2, \lambda_4\}$$

(3.2.1)

3.3 Pointers

It is considered that the storage capacity of each form of energy and for each system is known as soon as S_Λ is defined. [A.1] It remains to determine what is actually stored, and who is the owner of this energy. This is possible with P_Λ the *pointers*.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- E_Λ the energies of Λ
- P_Λ the pointers of Λ

Proposals

- $i \in E_\Lambda$ a form of energy
- $E \in \mathbb{R}_+^*$ an amount of energy of form i (J)
- $\lambda_1 \in \Lambda$ the system where E is located
- $\lambda_2 \in \Lambda$ the owner of E
- $P_\Lambda \subset \mathbb{R}_+^* \times E_\Lambda \times \Lambda \times \Lambda$ **(3.3.1)**
- $(E, i, \lambda_1, \lambda_2) \in P_\Lambda$ **(3.3.2)**

3.4 Conversions

Energy form conversions are performed by atomic systems. The conversion specifications vary from one system to another. It is the object C_Λ called *conversions* that will provide information on speed and efficiency of a conversion for such and such an energy form and for a given system.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- E_Λ the energies of Λ
- C_Λ the conversions of Λ

Proposals

- $(i, j) \in {E_\Lambda}^2$ two forms of energy
- $\lambda \in \Lambda$ an atomic system able to convert energy from a form i to a form j
- $\eta_\lambda^{i \to j} \in [0, 1]$ the efficiency of the conversion $i \to j$ in λ (1)
- $P_\lambda^{i \to j} \in \mathbb{R}_+$ the power of the conversion $i \to j$ in λ (W)
- $\tau_\lambda^{i \to j} \in \mathbb{R}_+$ the time delay of the conversion $i \to j$ in λ (s)
- $C_\Lambda \subset \Lambda \times E_\Lambda \times E_\Lambda \times [0, 1] \times \mathbb{R}_+ \times \mathbb{R}_+$ **(3.4.1)**
- $(\lambda, i, j, \eta_\lambda^{i \to j}, P_\lambda^{i \to j}, \tau_\lambda^{i \to j}) \in C_\Lambda$ **(3.4.2)**

3.5 Transfers

Energy flows from one atomic system to another. For this, transfer channels must exist. These channels are characterized by the systems they connect, the form of energy they transport, their efficiency, and speed of transmission. This information is gathered in T_Λ the *transfers*.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- E_Λ the energies of Λ
- T_Λ the transfers of Λ

Proposals

- $(\lambda_1, \lambda_2) \in \Lambda^2$ two atomic systems
- $i \in E_\Lambda$ a form of energy which can pass from λ_1 to λ_2 without being converted
- $\eta^i_{\lambda_1 \to \lambda_2} \in [0, 1]$ the efficiency of the transfer $\lambda_1 \to \lambda_2$ for the energy form i (1)
- $P^i_{\lambda_1 \to \lambda_2} \in \mathbb{R}_+$ the maximum flow of the transfer $\lambda_1 \to \lambda_2$ for the energy form i (W)
- $\tau^i_{\lambda_1 \to \lambda_2} \in \mathbb{R}_+$ the time delay of the transfer $\lambda_1 \to \lambda_2$ for the energy form i (s)
- $T_\Lambda \subset \Lambda \times \Lambda \times E_\Lambda \times [0, 1] \times \mathbb{R}_+ \times \mathbb{R}_+$ **(3.5.1)**
- $(\lambda_1, \lambda_2, i, \eta^i_{\lambda_1 \to \lambda_2}, P^i_{\lambda_1 \to \lambda_2}, \tau^i_{\lambda_1 \to \lambda_2}) \in T_\Lambda$ **(3.5.2)**

3.6 Register

For accounting and anticipation reasons, past, current and future transfers and conversions are memorized. This archiving is made possible by R_Λ the *register*. It only saves elementary operations (using only one transfer or conversion channel). The more complex operations (combining several energy transfers and conversions) are ordered sequences of these elementary operations.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- T_Λ the transfers of Λ
- C_Λ the conversions of Λ
- R_Λ the register of Λ

Proposals

- $c \in T_\Lambda \cup C_\Lambda$ the channel of transfer or conversion used
- $t_1 \in \mathbb{R}^*_+$ the date on which the emission begins (s)
- $t_2 \in \mathbb{R}^*_+$ the date on which the emission ends (s)
- $P \in \mathbb{R}^*_+$ the power of the emission (W)
- $R_\Lambda \subset (T_\Lambda \cup C_\Lambda) \times \mathbb{R}^*_+ \times \mathbb{R}^*_+ \times \mathbb{R}^*_+$ **(3.6.1)**
- $(c, t_1, t_2, P) \in R_\Lambda$ **(3.6.2)**

3.7 Laws

It only remains to define what is authorized in society and what is not. The formulation of these rules will be based on the set of mathematical objects introduced so far. These rules are logical propositions that are true or false. The object L_Λ called `laws` is a logical conjunction of these propositions.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- E_Λ the energies of Λ
- S_Λ the systems of Λ
- L_Λ the laws of Λ

Proposals

- $i \in E_\Lambda$ a form of energy that the society does not want to use as a primary source
- $\dot{W}^{i}_{\overline{\Lambda}\to\Lambda}$ the power supplied to Λ under the form i (W)
- $\Omega \subset \Lambda$ described by S_Λ and considered as vital
- F_Ω the Helmholtz free energy of Ω (J)
- $L_\Lambda \equiv (\dot{W}^{i}_{\overline{\Lambda}\to\Lambda} = 0) \wedge (\dot{F}_\Omega = 0)$ **(3.7.1)**

3.8 Synthesis

The mathematical objects exposed in this section are sufficient for the complete description of a society based on the principles established in the previous section.

Axiom

Hypotheses

- Λ the closed system corresponding to a sustainable society
- E_Λ the energies of Λ
- S_Λ the systems of Λ
- P_Λ the pointers of Λ
- C_Λ the conversions of Λ
- T_Λ the transfers of Λ
- R_Λ the register of Λ
- L_Λ the laws of Λ

Conclusion

- $(E_\Lambda, S_\Lambda, P_\Lambda, C_\Lambda, T_\Lambda, R_\Lambda, L_\Lambda)$ defines the society Λ **(3.8.1)**

These variables are limited in number and their form is standardized. It will therefore be possible to transmit them to a general program (the Bulk) that can orchestrate the functioning of the society described. This program will be able to operate societies with different political and economic models. And that is the objective of the project: to offer management tools based on the laws of physics and applicable to different configurations.

4 Optimization

The objects $(E_\Lambda, S_\Lambda, P_\Lambda, C_\Lambda, T_\Lambda, R_\Lambda, L_\Lambda)$ from the previous section provide a great deal of information about a society. This data must now be processed in order to operate the system. That is, to apply in the real world what is stated and manage unforeseen events. This requires decision-making algorithms. A very large number of possibilities are to be expected. To avoid the combinatorial explosion it will be necessary to adopt certain strategies which will be developed in the Bulk. This section only formalizes some of the optimization problems which will have to be solved.

4.1 Pathfinder

What is wanted here are the instructions which allow a quantity of energy to pass optimally from a form $i \in E_\Lambda$ in a system $\lambda_1 \in \Lambda$ to a form $j \in E_\Lambda$ in a system $\lambda_2 \in \Lambda$. Solving this problem requires using mathematical objects T_Λ and C_Λ. With the data provided by T_Λ it is possible to construct for each form of energy the graph of transfers. Here is an example for 5 atomic systems and 3 forms of energy. The vertices are the systems and the edges are the elements of T_Λ.

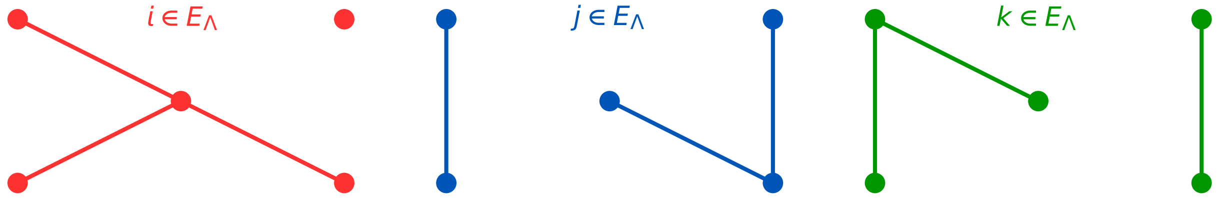


Figure 4.1.1

It is now possible to add the data provided by C_Λ. For this it is interesting to add a dimension corresponding to the different forms of energy. This results in a 3-dimensional graph which takes into account both the possibilities of exchange in space (for each form of energy) and the possibilities of conversion (for each atomic system). In such a graph, transfers and conversions will be put on an equal footing. Whether the edge is a transfer or a conversion, it is possible to associate the triplet of variables (η, τ, P) as defined with **(3.4.2)** and **(3.5.2)**

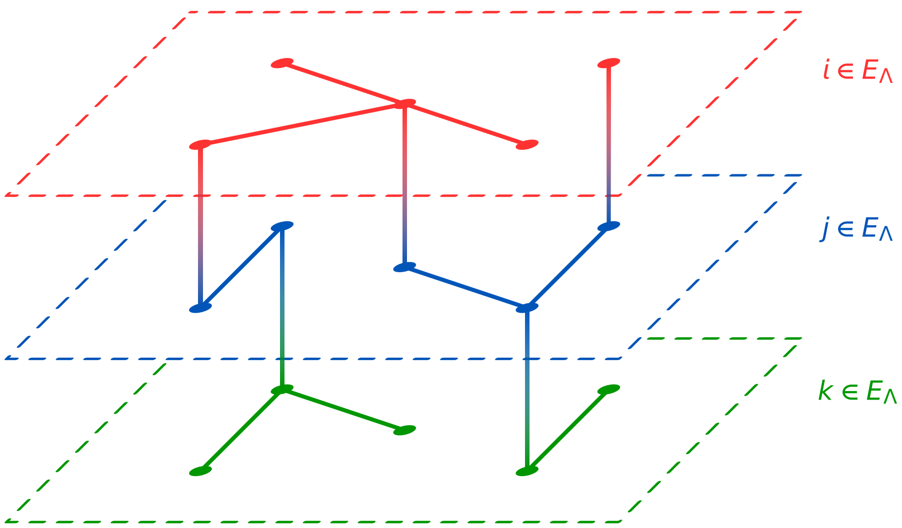


Figure 4.1.2

To such a graph it is possible to apply algorithms such as Dijkstra to find the optimal path. The weight of each edge will then be a function of (η, τ, P) defined according to the priorities of the user. (Efficiency, speed or flow.) Here the graph is undirected but only for the purpose of simplifying the representation. In practice, it will be oriented.

4.2 Equalizer

The previous problem only concerns transfers between two well-defined systems and forms of energy. A more general tool capable of managing the global distribution of energy is needed. That is to say for several transmitting systems and several receiving systems. For this we can consider the use of a flow algorithm such as Ford–Fulkerson.

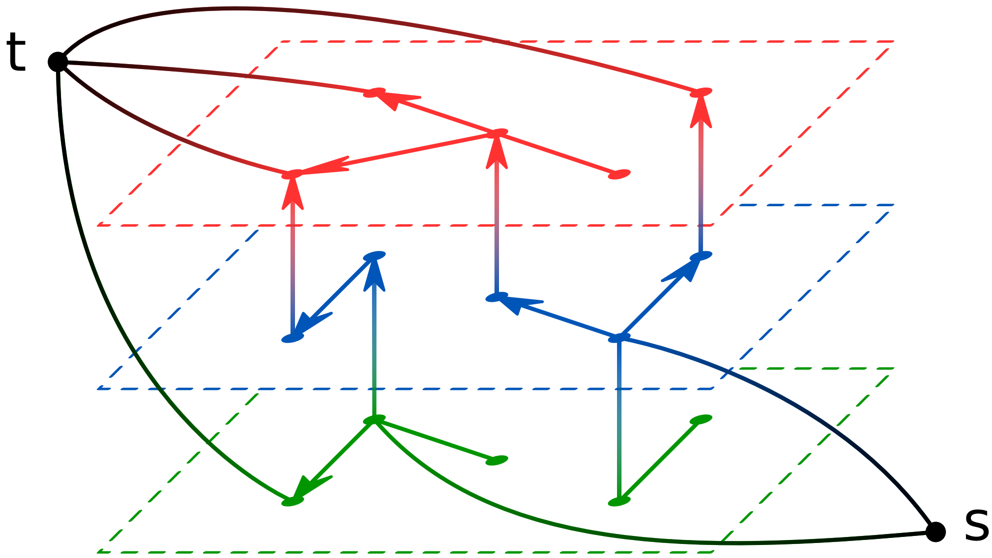


Figure 4.2.1

An improvement of this tool consists of adding a principle of equity. From the Ford–Fulkerson perspective, the solution of allocating 6 (arbitrary unity) to one system (that requires 6) and 4 to another (that also requires 6) is just as valid as the solution of allocating 5 to both. However, from a fair point of view, it would be preferable to choose the second solution. This principle of equity could identify with search for solution of **(4.2.2)**:

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- $T_{\overline{\Lambda}}\dot{S}^{C}_{\lambda}$ the instant consumption of the system $\lambda \in \Lambda$ (W)
- Π the set of possible power allocations
- $\dot{W}^{y}_{\lambda}$ the power allocated to the system $\lambda \in \Lambda$ in the allocation $y \in \Pi$ (W)
- $i \in \Pi$ a fair allocation

Proposals

- $$\sum_{\lambda \in \Lambda} \left(\frac{\dot{W}^{i}_{\lambda} - T_{\overline{\Lambda}}\dot{S}^{C}_{\lambda}}{T_{\overline{\Lambda}}\dot{S}^{C}_{\lambda}} \right)^2 = \min_{y \in \Pi} \left(\sum_{\lambda \in \Lambda} \left(\frac{\dot{W}^{y}_{\lambda} - T_{\overline{\Lambda}}\dot{S}^{C}_{\lambda}}{T_{\overline{\Lambda}}\dot{S}^{C}_{\lambda}} \right)^2 \right) \quad \textbf{(4.2.2)}$$

The tool should also favor solutions that maximize efficiency or minimize latency depending on user demand. For this, it might be relevant to implement the search of the augmenting paths in Ford–Fulkerson in the same way as the search in Dijkstra.

4.3 Advisor

Another optimization tool could be necessary to investigate the possibilities of improvements to the energy grid. It could use a space function that quantifies the severity of anomalies in an area of distribution. The maximums of this function, then correspond to areas with poor energy supplies, or suffering from faulty grid and which must be treated as a priority.

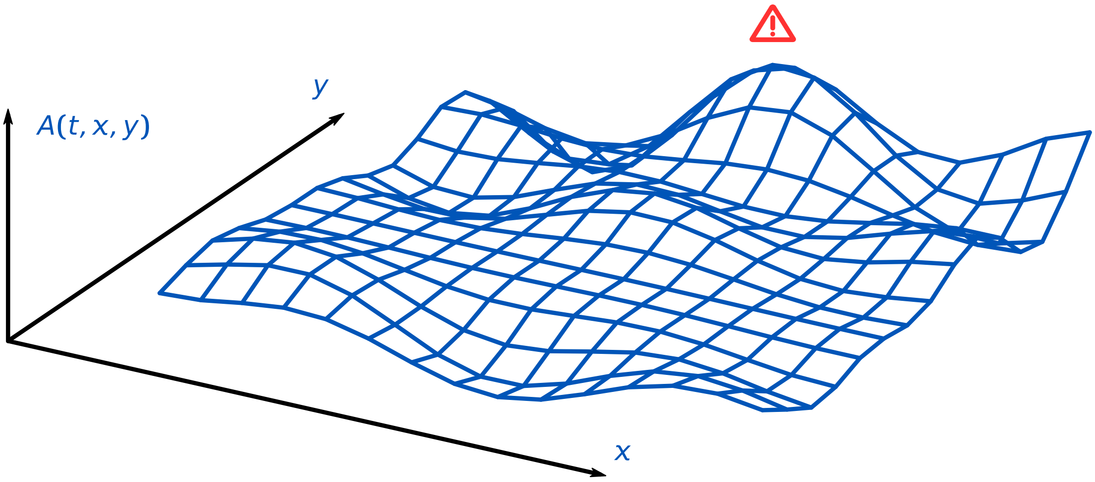


Figure 4.3.1

For instance, the maximums of the following function could be searched. It would remain to find if it is a problem of flow limited by the grid or an insufficiency in terms of the energy generated.

Example

Hypotheses

- Λ the closed system corresponding to a sustainable society
- $V(t,\vec{x}) \subset \Lambda$ the systems in the neighbourhood of the point $\vec{x}$ at time t
- $\dot{W}_\lambda$ the power allocated to the system $\lambda \in \Lambda$ (W)
- Δt the time period observed for averaging (s)

Proposals

- $$A(t,\vec{x}) = \frac{1}{\Delta t}\int_{t-\Delta t}^{t} \sum_{\lambda \in V(t,\vec{x})} T_{\overline{\Lambda}} \dot{S}_\lambda^C - \dot{W}_\lambda dt \quad \textbf{(4.3.1)}$$

The search for maximums can be carried out with non-linear optimization algorithms such as the conjugate gradient method or Newton's method. To this must be added an algorithm to define the actions to be taken once the location and the nature of the problem are identified.

4.4 Predictor

It would be relevant to have an algorithm capable of anticipating the instantaneous demand for energy. A neural network could be a good approach. This would require data to be stored over a period of time allowing certain cycles of consumption to be revealed.

4.5 Exchanger

Pointers will have a very specific purpose for citizens. They will become the new support for the currency. This will be deepened in the subsection 5.3 but what is already certain is that a program optimizing the exchanges of pointers will be necessary.

5 Application

The main principles are now established. This section shows how the objects $(E_\wedge, S_\wedge, P_\wedge, L_\wedge)$ can be defined. The goal is to propose an example society, which will serve as an initial proposal and which can be taken up in subsequent projects. This hypothetical society will be called `Foundation`. [A.5]

5.1 Energies

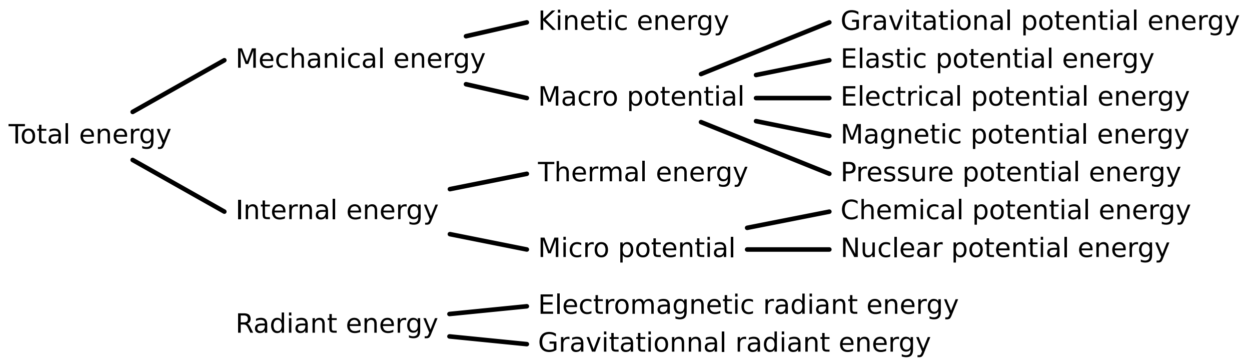


Figure 5.1.1

The description begins with the enumeration of forms of energy available in Foundation. This list will be based on the usual distinction between the different forms of energy in physics, illustrated above. Given the diversity of known chemical energy storage means and the different consequences they have on the environment, it is decided to distinguish a few major categories of these storage means.

Axiom

Hypotheses

- $\wedge$ the closed system corresponding to Foundation
- 01 refers to kinetic energy
- 02 refers to gravitational potential energy
- 03 refers to elastic potential energy
- 04 refers to electrical potential energy
- 05 refers to magnetic potential energy
- 06 refers to pressure potential energy
- 07 refers to thermal energy
- 08 refers to chemical energy stored in hydrocarbons
- 09 refers to chemical energy stored in the couple hydrogen / oxygen
- 10 refers to chemical energy stored in electric batteries
- 11 refers to chemical energy stored in biomass
- 12 refers to chemical energy stored in remaining means
- 13 refers to nuclear energy
- 14 refers to electromagnetic radiant energy

Conclusion

- $E_\wedge = [\![1, 14]\!]$ **(5.1.2)**

5.2 Systems

The next step is to define S_Λ the distinct subsystems of Foundation. This must be done while anticipating Ω, the vital ones (in order to facilitate the enunciation of laws). It is proposed here that Foundation must guarantee the following vital functions for its citizens: access to food, to medical care, to education, security, justice, political life, maintenance of public infrastructure, waste management and research for scientific progress. It is then reasonable to single out 9 systems in Ω intended to meet the 9 basic needs proposed.

Then it is possible to define the remaining components of society in order to complete Λ. It is chosen to single out the human beings, their homes, the private companies, the vehicles, the generator units, and energy distribution infrastructure.

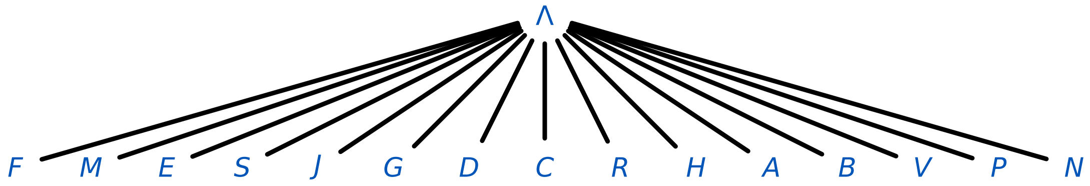
Axiom

Hypotheses

- Λ the closed system corresponding to Foundation
- $F \subset \Lambda$ the system ensuring access to food
- $M \subset \Lambda$ the system ensuring access to medical care
- $E \subset \Lambda$ the system ensuring access to education
- $S \subset \Lambda$ the system ensuring security
- $J \subset \Lambda$ the system ensuring justice
- $G \subset \Lambda$ the system ensuring government functions
- $D \subset \Lambda$ the system ensuring development and maintenance of public infrastructure
- $C \subset \Lambda$ the system ensuring life cycle of raw materials
- $R \subset \Lambda$ the system ensuring reasearch for scientific progress
- Ω the set of systems $\Sigma \subset \Lambda$ ensuring a vital function
- $H \subset \Lambda$ the human beings
- $A \subset \Lambda$ the private accommodation
- $B \subset \Lambda$ the private business
- $V \subset \Lambda$ the vehicles
- $P \subset \Lambda$ the power generator
- $N \subset \Lambda$ the nodes of the energy grid

Conclusion

- $\Omega = \{F, M, E, S, J, G, D, C, R\}$ **(5.2.1)**
- S_Λ corresponds to the following partitioning tree:



(5.2.2)

The 15 subsystems of Λ as described by **(5.2.2)** could have been partitioned again. But no need to go further for the moment. They will contain only atomic systems:

$$
\begin{aligned}
H &= \{ h \mid h \text{ is a human} \} \\
A &= \{ a \mid a \text{ is accommodation} \} \\
B &= \{ b \mid b \text{ is a business} \} \\
V &= \{ v \mid v \text{ is a vehicle} \} \\
P &= \{ p \mid p \text{ is a generator unit} \} \\
N &= \{ n \mid n \text{ is an energy grid node} \} \\
F &= \{ f \mid f \text{ is an agrifood production center} \} \\
M &= \{ m \mid m \text{ is an hospital} \} \\
E &= \{ e \mid e \text{ is a school} \} \\
S &= \{ s \mid s \text{ is a police or fire station} \} \\
J &= \{ j \mid j \text{ is a courthouse} \} \\
G &= \{ g \mid g \text{ is a government establishment} \} \\
D &= \{ d \mid d \text{ a civil engineering center} \} \\
C &= \{ c \mid c \text{ a center of treatment of raw materials} \} \\
R &= \{ r \mid r \text{ a research center} \}
\end{aligned}
$$

5.3 Pointers

For the vast majority of citizens, it would be inconvenient to store their capital in the form of real energy reserves at home. It would also be catastrophic from an efficiency point of view because too many unnecessary displacements of energy would be occasioned. This possession must therefore be dematerialized. The energy of a citizen would mainly remain in the energy grid N (where it can be transferred more efficiently) as long as he does not ask to recover it. The pointers P_Λ which already associate a location and a possessor with each amount of energy allow to formalize this.

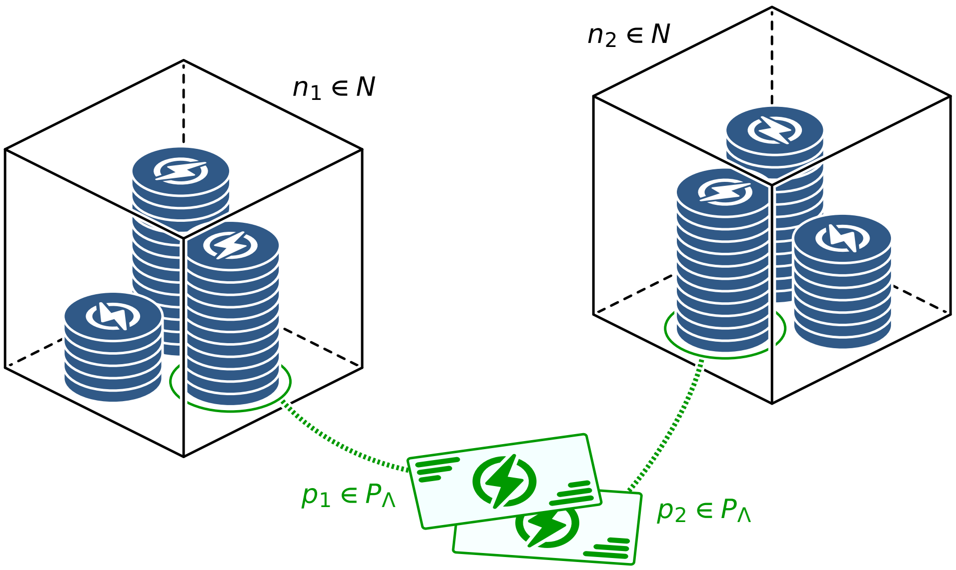


Figure 5.3.1

Given that a pointer refers to an amount of energy stored at a specific location, it does not have the same value anywhere in space. Indeed, it is necessary to subtract from the quantity of energy the losses linked to its transfer. Hence the interest of an algorithm (mentioned in the subsection 4.5) allowing optimal exchanges of pointers, (in the case where two citizens would exchange place of life for example).

Storing energy has an energy cost. It is not possible to offer unregulated storage capacity to citizens. It will therefore be necessary to establish storage fees. This aspect will be deepened in the next subsection with the equation **(5.4.1)**.

With this dematerialization there are now several ways to evaluate the amount of energy associated with an entity. The following equations allow to specify the owner, the place of storage and the form of the energy.

Example

Hypotheses

- Λ the closed system corresponding to Foundation
- E_Λ the energies as defined with **(5.1.2)**
- P_Λ the pointers of Foundation
- $i \in E_\Lambda$ a form of energy
- $\delta_\alpha^\beta = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases}$

Proposals

- $F_{\sigma_1}^{i,\sigma_2} = \sum_{\{E,j,\lambda_1,\lambda_2\} \in P_\Lambda} \delta_j^i \delta_{\lambda_1}^{\sigma_1} \delta_{\lambda_2}^{\sigma_2} E$ **(5.3.1)**
- $F_{\sigma_1}^{i} = \sum_{\sigma_2 \in \Lambda} F_{\sigma_1}^{i,\sigma_2}$ **(5.3.2)**
- $F^{i,\sigma_2} = \sum_{\sigma_1 \in \Lambda} F_{\sigma_1}^{i,\sigma_2}$ **(5.3.3)**
- $F_{\sigma_1}^{\sigma_2} = \sum_{i \in E_\Lambda} F_{\sigma_1}^{i,\sigma_2}$ **(5.3.4)**
- $F_{\sigma_1} = \sum_{\sigma_2 \in \Lambda} \sum_{i \in E_\Lambda} F_{\sigma_1}^{i,\sigma_2}$ **(5.3.5)**
- $F^{\sigma_2} = \sum_{\sigma_1 \in \Lambda} \sum_{i \in E_\Lambda} F_{\sigma_1}^{i,\sigma_2}$ **(5.3.6)**
- $\Delta F^{i,\sigma} = W^{i,\overline{\sigma} \to \sigma}$ **(5.3.7)**

Take-home message:

- **(5.3.1)** gives the amount of energy of form i in the system σ_1 possessed by σ_2.
- **(5.3.2)** gives the amount of energy of form i in the system σ_1.
- **(5.3.3)** gives the amount of energy of form i possessed by σ_2.
- **(5.3.4)** gives the amount of energy in the system σ_1 possessed by σ_2.
- **(5.3.5)** gives the amount of energy in the system σ_1.
- **(5.3.6)** gives the amount of energy possessed by σ_2.
- **(5.3.7)** implies that there is no loss in a change of owner.

It would be appreciated if the citizens did not have to worry about the forms of energies which are stored for them, nor of the place where they are located. The capital of a citizen will therefore be evaluated with the method **(5.3.6)**. The possibility of this simplification should be based on an average effect and will not concern the rest of the society (more focused on production and use of resources).

5.4 Laws

Here are defined some laws of Foundation. The list is not exhaustive and may of course be supplemented in the future. Some major guidelines are set, such as the establishment of a universal basic income and some constants and functions remain to be defined.

Example

Hypotheses

- Λ the closed system corresponding to Foundation
- E_Λ as defined with **(5.1.2)**
- Ω as defined with **(5.2.1)**
- S_Λ as defined with **(5.2.2)**
- $\delta_\alpha^\beta = \begin{cases} 1 & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases}$
- $F_N^h = \sum_{n \in N} F_n^h$ the amount of energy in N possessed by h (J)
- $C_{\text{UBI}} \in \mathbb{R}_+^*$ the universal basic income (W)
- $C_{\text{FEES}} \in \mathbb{R}_+^*$ energy storage costs, per joule (W/J)
- $\forall \Sigma \in \Omega$
  - $C_\Sigma \in \mathbb{R}_+^*$ the power required by a center of Σ per associated citizen (W)
  - $\forall \omega \in \Sigma \quad A(\omega) \subset H$ the citizens associated with the center ω (by geographic criteria)

Proposals

- $\forall h \in H \quad \dot{W}^{N \to h} = C_{UBI} - F_N^h C_{\text{FEES}}$ **(5.4.1)**
- $\forall \Sigma \in \Omega \quad \forall \omega \in \Sigma \quad \dot{W}_{N \to \omega} = C_\Sigma |A(\omega)|$ **(5.4.2)**
- $\dot{W}_{P \to N} = \sum_{\lambda \in \Lambda} \frac{\dot{W}_{N \to \lambda}}{\eta_{N \to \lambda}} + \sum_{h \in H} \dot{W}^{N \to h}$ **(5.4.3)**
- $\dot{W}^{8}_{\overline{\Lambda} \to \Lambda} = 0$ **(5.4.4)**
- $L_\Lambda \equiv (5.4.1) \wedge (5.4.2) \wedge (5.4.3) \wedge (5.4.4)$ **(5.4.5)**

Take-home message:

- **(5.4.1)** guarantees a UBI for each citizen and defines the storage conditions.
- **(5.4.2)** guarantees an adequate supply of energy for essential services.
- **(5.4.3)** includes distribution-related losses in the calculation of the power required by the society.
- **(5.4.4)** prohibits the use of fossil fuels as primary sources.

5.5 About matter

The system Λ is considered closed since its introduction. This does not prevent the exchange of matter inside, between the atomic systems. Nothing special in this regard. Suppliers like C make available raw materials exchangeable for energy, or pointers.

6 Interest

The project is now presented, at least in its major aspects. However, the question of its relevance has not yet been addressed. This section presents the motivation behind the development of Entropy Economy System and the benefits that are expected from it.

6.1 Simplify, disambiguate, condense

What is stated with an equation is much less subject to misinterpretation. Entropy Economy System proposes the elementary bricks from which to build these equations. It allows a more global vision of the construction of a society and a faster access to its key elements.

6.2 Standardize, generalize, unify

The language of physics is powerful and shared all over the world. The project extends this existing tool to the description of human societies. Rigor of reasoning and valid elementary principles make it possible to co-develop a project as complex as the structuring of a society.

6.3 Simulate, compare, study

The ability to translate mathematically a wide variety of economic, political and social orientations is an essential aspect of the project. Standardizing the description tools allows the simulation and comparison of different configurations. The most relevant models can then be highlighted.

6.4 Rationalize, protect, conciliate

This point is the most important. Nothing known at our scale escapes the laws that govern the universe and that physics is trying to model. This new formalism may simplify the description of a society, but it is above all a means of understanding what is possible and what is not. Try to build economic laws that do not care about their physical implications and nature will remind you that your model is not consistent. Instead of witnessing the failure of what could seem economically viable, it would be better to take into account as soon as possible these main principles that rule our world.

A Notes

A.1 System properties

The definition of an atomic system σ implies the definition of the following properties:

- Its location $\sigma_L \in \mathbb{R}^n$ with n the dimension of space.
- Its storage capacity $\sigma_E(i) \in \mathbb{R}_+$ for a form of energy $i \in E_\Lambda$. [3.1]

It is possible to generalize these properties to a non-atomic system Σ. For example:

- $\Sigma_L = \sum_{\sigma \in \Sigma} w \sigma_L$ with $w \in [0,1]$ a weighting coefficient such as $\frac{\sigma_E(i)}{\Sigma_E(i)}$ with $i \in E_\Lambda$
- $\Sigma_E(i) = \sum_{\sigma \in \Sigma} \sigma_E(i)$

A.2 Helmholtz free energy in a sustainable system

What is implicitly expected from a "sustainable" system Σ is its ability, at any time, to produce work. If F_Σ is allowed to decrease, it can happen that $F_\Sigma = 0$, which prevents any work from being produced. To prevent this, we simply declare that F_Σ must be constant (or increasing, but that is not the problem).

A.3 Surroundings temperature

Originally the hypothesis $dT_{\overline{\Lambda}} = 0$ was a way to simplify the equation **(2.1.1)**. But this does not preclude finding a physical explanation. It is indeed unlikely that a society will one day settle in a region of space where the global temperature experiences dramatic variations. And then, if $dT_{\overline{\Lambda}} = 0$ allows a good modeling of the society over a short period of time such as a few minutes, nothing prevents from reconsidering everything with a different temperature $T_{\overline{\Lambda}}$ a few hours or days later.

The surroundings temperature $T_{\overline{\lambda}}$ of a component $\lambda \in \Lambda$ is voluntarily assimilated to the surroundings temperature $T_{\overline{\Lambda}}$ of the society. This is actually due to the way the system Λ is defined. It is possible to illustrate this choice with the drawing below.

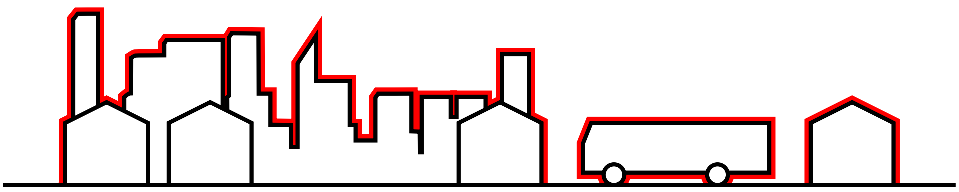


Figure A.3.1

The red contours correspond to the surfaces of Λ. Considering λ the bus, it comes naturally $T_{\overline{\lambda}} = T_{\overline{\Lambda}}$.

A.4 Systems and singletons

It is possible to confuse σ and $\{\sigma\}$. An atomic system is a system. Not the opposite.

A.5 Foundation

The section 5 that defines some aspects of Foundation is proposed as an example of application only. Foundation is not presented as a preferable model to another to date. Its configuration will first be studied and improved by the Bulk. Then, at this time, it will be offered as a tested model. And only from this moment, the economic, social and political models of Foundation would be recommended by the Pale Blueprint project.

B Sources

Besides the briefly mentioned optimization methods, this article is based on general notions of thermodynamics. The proposed sources are given as additional information. Any other standard course in thermodynamics would introduce the concepts used just as well.

- 1.1 Physical systems
  - https://en.wikipedia.org/wiki/Physical_system
- 1.2 Physical quantities
  - https://en.wikipedia.org/wiki/Physical_quantity
- 1.3 First law of thermodynamics
  - https://en.wikipedia.org/wiki/First_law_of_thermodynamics
  - https://en.wikipedia.org/wiki/Internal_energy
- 1.4 Second law of thermodynamics
  - https://en.wikipedia.org/wiki/Second_law_of_thermodynamics
  - https://en.wikipedia.org/wiki/Entropy_(classical_thermodynamics)
- 2.1 General equation
  - https://en.wikipedia.org/wiki/Helmholtz_free_energy
- 3.4 Conversions
  - https://en.wikipedia.org/wiki/Energy_transformation
- 4.1 Pathfinder
  - https://en.wikipedia.org/wiki/Dijkstra's_algorithm
- 4.2 Equalizer
  - https://en.wikipedia.org/wiki/Ford-Fulkerson_algorithm
- 4.3 Advisor
  - https://en.wikipedia.org/wiki/Gradient_method
- 5.1 Energies
  - https://en.wikipedia.org/wiki/Mechanical_energy
  - https://en.wikipedia.org/wiki/Internal_energy
  - https://en.wikipedia.org/wiki/Radiant_energy